Review 5.1-5.5

(S book)...

Exer. 1–2: Find $f^{-1}(x)$.

1 $f(x) = 10 - 15x$

2 $f(x) = 9 - 2x^2, \quad x \le 0$

Exer. 3–4: Show that the function f has an inverse function, and find $[D_x f^{-1}(x)]_{x=a}$ for the given number a.

3 $f(x) = 2x^3 - 8x + 5, \quad -1 \le x \le 1; \quad a = 5$

4 $f(x) = e^{3x} + 2e^x - 5, \quad x \ge 0; \quad a = -2$

Exer. 5–38: Find $f'(x)$ if $f(x)$ is the given expression.

5 $\ln|4 - 5x^3|^5$

6 $\ln|x^2 - 7|^3$

7 $(1 - 2x)\ln|1 - 2x|$

8 $\log\left|\dfrac{2 - 9x}{1 - x^2}\right|$

9 $\ln\dfrac{(3x+2)^4\sqrt{6x-5}}{8x-7}$

10 $\ln\sqrt[4]{\dfrac{x}{3x+5}}$

11 $\dfrac{1}{\ln(2x^2+3)}$

12 $\dfrac{\ln x}{e^{2x}+1}$

13 $\dfrac{x}{\ln x}$

14 $\dfrac{\ln x}{x}$

15 $e^{\ln(x^2+1)}$

16 $\ln e^{\sqrt{x}}$

17 $\ln(e^{4x}+9)$

18 $4^{\sqrt{2x+3}}$

19 $10^x \log x$

20 $5^{3x} + (3x)^5$

21 $\sqrt{\ln\sqrt{x}}$

22 $(1 + \sqrt{x})^e$

23 $x^2 e^{-x^2}$

24 $\dfrac{2^{-3x}}{x^3+4}$

25 $\sqrt{e^{3x} + e^{-3x}}$

26 $(x^2+1)^{2x}$

27 $10^{\ln x}$

28 $7^{\ln|x|}$

29 $x^{\ln x}$

30 $(\ln x)^{\ln x}$

31 $\ln|\tan x - \sec x|$

32 $\ln\csc\sqrt{x}$

33 $\csc e^{-2x}\cot e^{-2x}$

34 $x^2 e^{\tan 2x}$

35 $\ln\cos^4 4x$

36 $3^{\sin 3x}$

37 $(\sin x)^{\cos x}$

38 $\dfrac{1}{\sin^2 e^{-3x}}$

Exer. 39–40: Use implicit differentiation to find y'.

39 $1 + xy = e^{xy}$

40 $\ln(x+y) + x^2 - 2y^3 = 1$

Exer. 41–42: Use logarithmic differentiation to find dy/dx.

41 $y = (x+2)^{4/3}(x-3)^{3/2}$

42 $y = \sqrt[3]{(3x-1)\sqrt{2x+5}}$

Exer. 43–78: Evaluate the integral.

43 (a) $\displaystyle\int \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx$ (b) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}\, e^{\sqrt{x}}}\, dx$

44 (a) $\displaystyle\int e^{-3x+2}\, dx$ (b) $\displaystyle\int_0^1 e^{-3x+2}\, dx$

45 (a) $\displaystyle\int x4^{-x^2}\, dx$ (b) $\displaystyle\int_0^1 x4^{-x^2}\, dx$

46 (a) $\displaystyle\int \frac{x^2+1}{x^3+3x}\, dx$ (b) $\displaystyle\int_1^2 \frac{x^2+1}{x^3+3x}\, dx$

47 $\displaystyle\int x\tan x^2\, dx$

48 $\displaystyle\int \cot\left(x + \frac{\pi}{6}\right) dx$

49 $\displaystyle\int x^e\, dx$

50 $\displaystyle\int \frac{1}{7-5x}\, dx$

51 $\displaystyle\int \frac{1}{x - x\ln x}\, dx$

52 $\displaystyle\int \frac{1}{x\ln x}\, dx$

53 $\displaystyle\int \frac{(1+e^x)^2}{e^{2x}}\, dx$

54 $\displaystyle\int \frac{(e^{2x}+e^{3x})^2}{e^{5x}}\, dx$

55 $\displaystyle\int \frac{x^2}{x+2}\, dx$

56 $\displaystyle\int \frac{x^2+1}{x+1}\, dx$

57 $\displaystyle\int \frac{e^{4/x^2}}{x^3}\, dx$

58 $\displaystyle\int \frac{e^{1/x}}{x^2}\, dx$

59 $\displaystyle\int \frac{x}{x^4+2x^2+1}\, dx$

60 $\displaystyle\int \frac{5x^3}{x^4+1}\, dx$

61 $\int \frac{e^x}{1+e^x}\,dx$

62 $\int (1+e^{-3x})^2\,dx$

63 $\int 5^x e^x\,dx$

64 $\int x10^{(x^2)}\,dx$

65 $\int \frac{1}{x\sqrt{\log x}}\,dx$

66 $\int 7^x\sqrt{1+7^x}\,dx$

67 $\int e^{-x}\sin e^{-x}\,dx$

68 $\int \tan x\, e^{\sec x}\sec x\,dx$

69 $\int \frac{\csc^2 x}{1+\cot x}\,dx$

70 $\int \frac{\cos x+\sin x}{\sin x-\cos x}\,dx$

71 $\int \frac{\cos 2x}{1-2\sin 2x}\,dx$

72 $\int 3^x(3+\sin 3^x)\,dx$

73 $\int e^x\tan e^x\,dx$

74 $\int \frac{\sec(1/x)}{x^2}\,dx$

75 $\int (\csc 3x+1)^2\,dx$

76 $\int \cos 2x\csc 2x\,dx$

77 $\int (\cot 9x+\csc 9x)\,dx$

78 $\int \frac{\sin x+1}{\cos x}\,dx$

79 Solve the differential equation $y''=-e^{-3x}$ subject to the conditions $y=-1$ and $y'=2$ if $x=0$.

80 In *seasonal population growth*, the population $q(t)$ at time t (in years) increases during the spring and summer but decreases during the fall and winter. A differential equation that is sometimes used to describe this type of growth is $q'(t)/q(t)=k\sin 2\pi t$, where $k>0$ and $t=0$ corresponds to the first day of spring.

(a) Show that the population $q(t)$ is seasonal.

(b) If $q_0=q(0)$, find a formula for $q(t)$.

81 A particle moves on a coordinate line with an acceleration at time t of $e^{t/2}$ cm/sec^2. At $t=0$ the particle is at the origin and its velocity is 6 cm/sec. How far does it travel during the time interval $[0,4]$?

82 Find the local extrema of $f(x)=x^2\ln x$ for $x>0$. Discuss concavity, find the points of inflection, and sketch the graph of f.

83 Find an equation of the tangent line to the graph of the equation $y=xe^{1/x^3}+\ln|2-x^2|$ at the point $P(1,e)$.

84 Find the area of the region bounded by the graphs of the equations $y=e^{2x}$, $y=x/(x^2+1)$, $x=0$, and $x=1$.

85 The region bounded by the graphs of $y=e^{4x}$, $x=-2$, $x=-3$, and $y=0$ is revolved about the x-axis. Find the volume of the resulting solid.

86 The 1980 population estimate for India was 651 million, and the population has been increasing at a rate of about 2% per year, with the rate of increase proportional to the number of people. If t denotes the time (in years) after 1980, find a formula for $N(t)$, the population (in millions) at time t. Assuming that this rapid growth rate continues, estimate the population and the rate of population growth in the year 2000.

87 A radioactive substance has a half-life of 5 days. How long will it take for an amount A to disintegrate to the extent that only 1% of A remains?

88 The carbon-14 dating equation $T=-8310\ln x$ is used to predict the age T (in years) of a fossil in terms of the percentage $100x$ of carbon still present in the specimen (see Exercise 19, Section 7.6).

(a) If $x=0.04$, estimate the age of the fossil to the nearest 1000 years.

(b) If the maximum error in estimating x in part (a) is ± 0.005, use differentials to approximate the maximum error in T.

89 The rate at which sugar dissolves in water is proportional to the amount that remains undissolved. Suppose that 10 pounds of sugar are placed in a container of water at 1:00 P.M., and one-half is dissolved at 4:00 P.M.

(a) How long will it take two more pounds to dissolve?

(b) How much of the 10 pounds will be dissolved at 8:00 P.M.?

90 According to Newton's law of cooling, the rate at which an object cools is directly proportional to the difference in temperature between the object and its surrounding medium. If $f(t)$ denotes the temperature at time t, show that $f(t)=T+[f(0)-T]e^{-kt}$, where T is the temperature of the surrounding medium and k is a positive constant.

91 The bacterium *E. coli* undergoes cell division approximately every 20 minutes. Starting with 100,000 cells, determine the number of cells after 2 hours.

92 The differential equation $p\,dv+cv\,dp=0$ describes the adiabatic change of state of air for pressure p, volume v, and a constant c. Solve for p as a function of v.