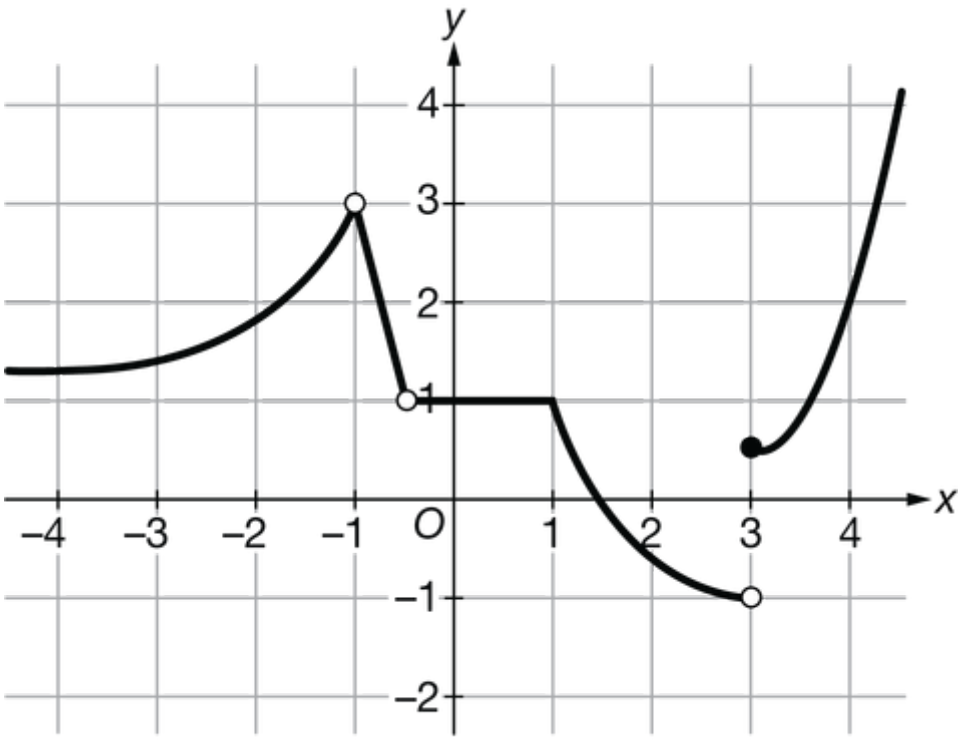
Unit 1 Review

1.



Graph of f

The graph of the function f is shown above. What is $\lim_{x \to -1} f(f(x))$?

(A) 3

(B) $\frac{1}{2}$

(C) -1 ✓

(D) nonexistent

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2. $$f(x) = \begin{cases} x^2 + 2x & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ x^3 + x^2 + x & \text{for } 1 < x < 3 \\ 0 & \text{for } x = 3 \\ 2x + 1 & \text{for } x > 3 \end{cases}$$

Let f be the piecewise function defined above. Which of the following statements is false?

(A) f is continuous at $x = 1$.

(B) f is continuous at $x = 2$.

(C) f is continuous at $x = 3$. ✔

(D) f is continuous at $x = 4$.

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3. Let f be the function given by $f(x) = \frac{2x^2+14x-16}{x^2-9x+8}$. For what values of **x** does **f** have a removable discontinuity?

(A) 1 only ✔

(B) 8 only

(C) -8 and 1

(D) 1 and 8

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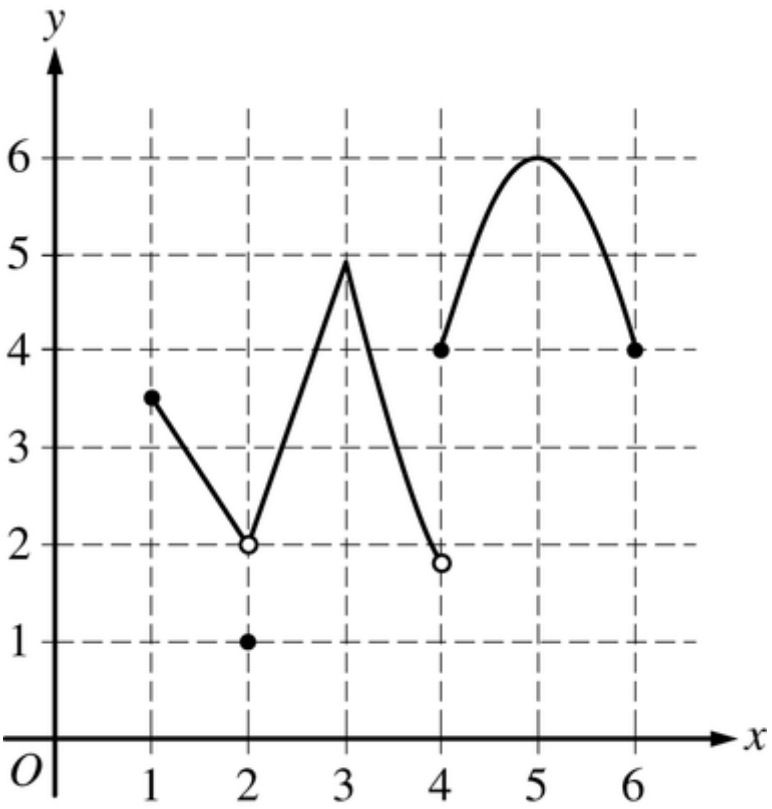
4.



Graph of f

The graph of the function **f** is shown above. Which of the following statements is false?

(A) $\lim_{x \to 2} f(x)$ exists.

(B) $\lim_{x \to 3} f(x)$ exists.

(C) $\lim_{x \to 4} f(x)$ exists. ✔

(D) $\lim_{x \to 5} f(x)$ exists.

(E) The function **f** is continuous at $x = 3$.

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5. NO CALCULATOR IS ALLOWED FOR THIS QUESTION.

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Show all of your work, even though the question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

$$f(x) = \begin{cases} \frac{2x^2+5x-3}{x^2+4x+3} & \text{for } x < -3 \\ kx + \frac{1}{2} & \text{for } -3 \le x \le 0 \\ \frac{2^x}{3^x-1} & \text{for } x > 0 \end{cases}$$

Let f be the function defined above, where k is a constant.

(a) For what value of k, if any, is f continuous at $x = -3$? Justify your answer.

(b) What type of discontinuity does f have at $x = 0$? Give a reason for your answer.

(c) Find all horizontal asymptotes to the graph of f. Show the work that leads to your answer.

Please respond on separate paper, following directions from your teacher.

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Part A

At most 2 out of 3 points if mathematical notation for limits is missing or incorrect.

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	2	3 ✔

The student response accurately includes all three of the criteria below.

Solution:

$$\lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \frac{2x^2+5x-3}{x^2+4x+3} = \lim_{x\to -3^-} \frac{(2x-1)(x+3)}{(x+1)(x+3)}$$

$$= \lim_{x\to -3^-} \frac{2x-1}{x+1} = \frac{\lim\limits_{x\to -3^-}(2x-1)}{\lim\limits_{x\to -3^-}(x+1)}$$

$$= \frac{2(-3)-1}{-3+1} = \frac{7}{2}$$

$$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} \left(kx+\frac{1}{2}\right) = -3k+\frac{1}{2}$$

$$f(-3) = -3k+\frac{1}{2}$$

f is continuous at $x=-3$ if $\lim\limits_{x\to -3^-} f(x) = \lim\limits_{x\to -3^+} f(x) = f(-3)$.

$$-3k+\frac{1}{2} = \frac{7}{2} \;\Rightarrow\; -3k = 3 \;\Rightarrow\; k = -1$$

Part B

Select a point value to view scoring criteria, solutions, and/or examples and to score the response.

0	1	✓ 2

he student response accurately includes both of the criteria below.

Solution:

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{2^x}{3^x - 1} = \infty$ because the numerator approaches 1, while the denominator approaches 0 from the right.

Therefore, f has a discontinuity due to a vertical asymptote at $x = 0$.

Part C

At most 1 out of 2 points if both correct asymptotes are listed without reference to limits OR if one correct asymptote is listed without any incorrect asymptotes and/or reference to limits.

Students do not have to show the factorization to earn the second point.

Select a point value to view scoring criteria, solutions, and/or examples to score the response.

0	1	✓ 2

The student response accurately includes both of the criteria below.

Solution:

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{2^x}{3^x - 1} = \lim_{x \to \infty} \frac{2^x}{3^x} = \lim_{x \to \infty} \left(\frac{2}{3}\right)^x = 0$ because $0 < \frac{2}{3} < 1$.

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$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{2x^2+5x-3}{x^2+4x+3} = \lim_{x\to-\infty} \frac{x^2\left(2+\frac{5}{x}-\frac{3}{x^2}\right)}{x^2\left(1+\frac{4}{x}+\frac{3}{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{2+\frac{5}{x}-\frac{3}{x^2}}{1+\frac{4}{x}+\frac{3}{x^2}} = \frac{\lim_{x\to-\infty}\left(2+\frac{5}{x}-\frac{3}{x^2}\right)}{\lim_{x\to-\infty}\left(1+\frac{4}{x}+\frac{3}{x^2}\right)}$$

$$= \frac{2+0-0}{1+0+0} = 2$$

The graph of f has horizontal asymptotes at $y = 0$ and $y = 2$.

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6. For which of the following does $\lim_{x\to\infty} f(x) = 0$?

I. $f(x) = \frac{\ln x}{x^{99}}$

II. $f(x) = \frac{e^x}{\ln x}$

III. $f(x) = \frac{x^{99}}{e^x}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only ✓

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7. Let g and h be the functions defined by $g(x) = -x^2 - 2x + 3$ and $h(x) = \frac{1}{2}x^2 + x + \frac{13}{2}$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for all x, what is $\lim_{x\to-1} f(x)$?

(A) 4

(B) 5

(C) 6

(D) The limit cannot be determined from the information given. ✔

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8. Let g and h be the functions defined by $g(x) = \sin\left(\frac{\pi}{2}x\right) + 4$ and $h(x) = -\frac{1}{4}x^3 + \frac{3}{4}x + \frac{9}{2}$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for $-1 < x < 2$, what is $\lim_{x \to 1} f(x)$?

(A) 4

(B) $\frac{9}{2}$

(C) 5 ✔

(D) The limit cannot be determined from the information given.

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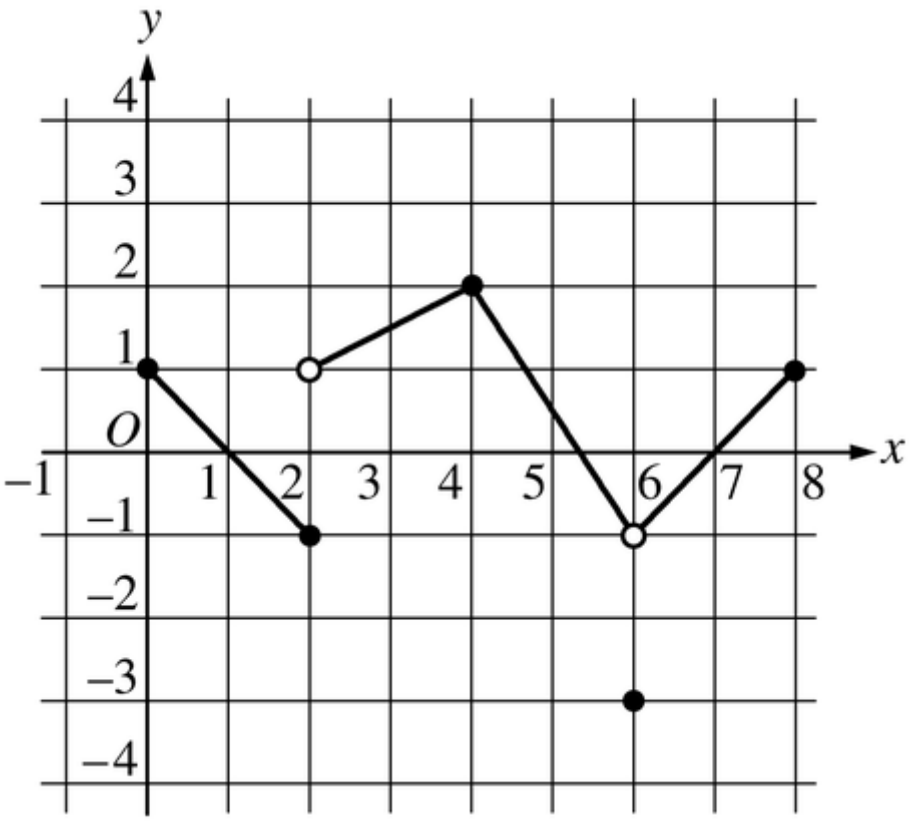
9.



The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim_{x \to 2^-} f(x) = f(2)$

II. $\lim_{x \to 6^-} f(x) = \lim_{x \to 6^+} f(x)$

III. $\lim_{x \to 6} f(x) = f(6)$

(A) II only

(B) III only

(C) I and II only ✓

(D) II and III only

(E) I, II, and III

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10. Which of the following limits are equal to −1 ?

1. $\lim_{x \to 0^-} \frac{|x|}{x}$
2. $\lim_{x \to 3} \frac{x^2 - 7x + 12}{3 - x}$
3. $\lim_{x \to \infty} \frac{1 - x}{1 + x}$

(A) I only

(B) I and III only ✔

(C) II and III only

(D) I, II, and III only

11. What are the equations of the horizontal asymptotes of the graph of $y = \frac{2}{\sqrt{x^2 - 1}}$?

(A) y = 0 only

(B) y = 1 only

(C) y = 2 only

(D) y = -2 only and y = 2 only ✔

(E) y = -1 only and y = 1 only

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12. Let f be the function defined by $f(x) = \frac{x^4-4x^2}{x^2-4x}$. Which of the following statements is true?

(A) f has a discontinuity due to a vertical asymptote at $x = 0$ and at $x = 4$.

(B) f has a removable discontinuity at $x = 0$ and a jump discontinuity at $x = 4$.

(C) f has a removable discontinuity at $x = 0$ and a discontinuity due to a vertical asymptote at $x = 4$. ✔

(D) f is continuous at $x = 0$, and f has a discontinuity due to a vertical asymptote at $x = 4$.

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13. $f(x) = \begin{cases} \frac{x^2-7x+10}{b(x-2)} & \text{for } x \neq 2 \\ b & \text{for } x = 2 \end{cases}$

Let **f** be the function defined above. For what value of **b** is **f** continuous at $x = 2$?

(A) -3

(B) $\sqrt{2}$

(C) 3

(D) 5

(E) There is no such value of **b**. ✔

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14. If the function f is continuous for all real numbers and if $f(x) = \frac{x^2-4}{x+2}$ when $x \neq -2$, then $f(-2) =$

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(A) -4 ✓

(B) -2

(C) -1

(D) 0

(E) 2

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15. If f is the function defined by $f(x) = \frac{x^2-4}{\sqrt{x}-\sqrt{2}}$, then $\lim_{x \to 2} f(x)$ is equivalent to which of the following?

(A) $\lim_{x \to 2} (x+2)\left(\sqrt{x}+\sqrt{2}\right)$ ✓

(B) $\lim_{x \to 2} \left(\sqrt{x}+\sqrt{2}\right)$

(C) $\lim_{x \to 2} \left(x\sqrt{x}-2\sqrt{2}\right)$

(D) $\lim_{x \to 2} (x+2)\left(x^2+4\right)$

16.

x	-2	-1	0	1	2	3
$f(x)$	-2	5	2	-4	-1	3

Selected values of a continuous function f are given in the table above. What is the fewest possible number of zeros of f in the interval $[-2, 3]$?

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(A) Zero

(B) One

(C) Two

(D) Three ✓

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17. Let **f** be the function defined by $f(x) = \begin{cases} x^2 + 2 & \text{for } x \leq 3, \\ 6x + k & \text{for } x > 3. \end{cases}$

If **f** is continuous at $x = 3$, what is the value of **k** ?

(A) -7 ✓

(B) 2

(C) 3

(D) 7

(E) There is no such value of **k**.

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18. If the graph of $y = \frac{ax+b}{x+c}$ has a horizontal asymptote **y**=2 and a vertical asymptote x=-3, then **a+c=**

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(A) -5

(B) -1

(C) 0

(D) 1

(E) 5 ✔

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19. $f(x) = \begin{cases} \frac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$

Let **f** be the function defined above. For what value of **k** is **f** continuous at $x = 2$?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 5 ✔

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20.

$f(3) = 3$	$\lim_{x \to 3} f(x) = 2$
$g(3) = 8$	$\lim_{x \to 3} g(x) = 8$
$h(3) = 4$	$\lim_{x \to 3} h(x) = 2$

The table above gives selected values and limits of the functions f, g, and h. What is $\lim_{x \to 3} (h(x)(2f(x) + 3g(x)))$?

(A) 120

(B) 104

(C) 56 ✓

(D) 32

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21. The function g is continuous at all x except $x = 2$. If $\lim_{x \to 2} g(x) = \infty$, which of the following statements about g must be true?

(A) $g(2) = \infty$

(B) The line $x = 2$ is a horizontal asymptote to the graph of g.

(C) The line $x = 2$ is a vertical asymptote to the graph of g. ✓

(D) The line $y = 2$ is a vertical asymptote to the graph of g.

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22. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

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(A) $y = \frac{\sin(5x)}{x}$

(B) $y = 5x$

(C) $y = \frac{1}{x-5}$

(D) $y = \frac{5x}{1-x}$

(E) $y = \frac{20x^2-x}{1+4x^2}$ ✓

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23. Let **f** be the function given by $f(x) = \frac{x-2}{2|x-2|}$. Which of the following is true?

(A) $\lim_{x \to 2} f(x) = \frac{1}{2}$

(B) **f** has a removable discontinuity at **x = 2.**

(C) **f** has a jump discontinuity at **x = 2 .** ✓

(D) **f** has a discontinuity due to a vertical asymptote at **x = 2.**

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24. Which of the following functions are continuous on the interval $0 < x < 5$?

1. $f(x) = \frac{x-3}{x^2-9}$
2. $g(x) = \frac{x-3}{x^2+9}$
3. $h(x) = \ln(x-3)$

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(A) II only ✓

(B) I and II only

(C) I and III only

(D) II and III only

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25. Let g and h be the functions defined by $g(x) = \sin\left(\frac{\pi}{2}(x+2)\right) + 3$ and $h(x) = -\frac{1}{4}x^3 - \frac{3}{2}x^2 - \frac{9}{4}x + 3$. If f is a function that satisfies $g(x) \le f(x) \le h(x)$ for $-2 < x < 0$, what is $\lim_{x \to -1} f(x)$?

(A) 3

(B) 3.5

(C) 4 ✓

(D) The limit cannot be determined from the information given.

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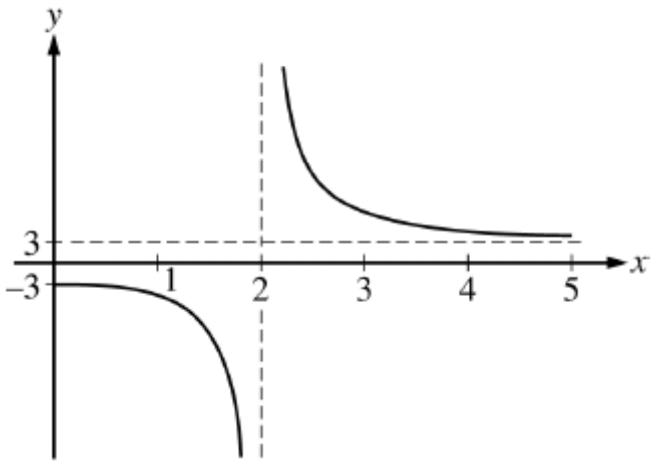
26.



The function **f** is given by $f(x) = \frac{ax^2+12}{x^2+b}$. The figure above shows a portion of the graph of **f** . Which of the following could be the values of the constants a and b?

(A) a = –3, b = 2

(B) a = 2, b = –3

(C) a = 2, b = –2

(D) a = 3, b = –4 ✔

(E) a = 3, b = 4

27.

x	0	1	2
$f(x)$	1	k	2

The function **f** is continuous on the closed interval [0,2] and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval [0,2] if **k=**

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(A) 0 ✓

(B) $\frac{1}{2}$

(C) 1

(D) 2

(E) 3