AP Calculus - Rotational Volume Practice (6.2 SWOK)

 Must show set-up + then #5-33 odd

Can use calculator

Exer. 5–24: Sketch the region R bounded by the graphs of the equations, and find the volume of the solid generated if R is revolved about the indicated axis.

5 $y = 1/x$, $x = 1$, $x = 3$, $y = 0$; x-axis

6 $y = \sqrt{x}$, $x = 4$, $y = 0$; x-axis

7 $y = x^2 - 4x$, $y = 0$; x-axis

8 $y = x^3$, $x = -2$, $y = 0$; x-axis

9 $y = x^2$, $y = 2$; y-axis

10 $y = 1/x$, $y = 1$, $y = 3$, $x = 0$; y-axis

11 $x = 4y - y^2$, $x = 0$; y-axis

12 $y = x$, $y = 3$, $x = 0$; y-axis

13 $y = x^2$, $y = 4 - x^2$; x-axis

14 $x = y^3$, $x^2 + y = 0$; x-axis

15 $y = x$, $x + y = 4$, $x = 0$; x-axis

16 $y = (x-1)^2 + 1$, $y = -(x-1)^2 + 3$; x-axis

17 $y^2 = x$, $2y = x$; y-axis

18 $y = 2x$, $y = 4x^2$; y-axis

19 $x = y^2$, $x - y = 2$; y-axis

? $+ y = 1$, $x - y = -1$, $x = 2$; y-axis

$= \sin 2x$, $x = 0$, $x = \pi$, $y = 0$; x-axis

22 $y = 1 + \cos 3x$, $x = 0$, $x = 2\pi$, $y = 0$; x-axis

23 $y = \sin x$, $y = \cos x$, $x = 0$, $x = \pi/4$; x-axis

24 $y = \sec x$, $y = \sin x$, $x = 0$, $x = \pi/4$; x-axis

Exer. 25–26: Sketch the region R bounded by the graphs of the equations, and find the volume of the solid generated if R is revolved about the given line.

25 $y = x^2$, $y = 4$

(a) $y = 4$ (b) $y = 5$

(c) $x = 2$ (d) $x = 3$

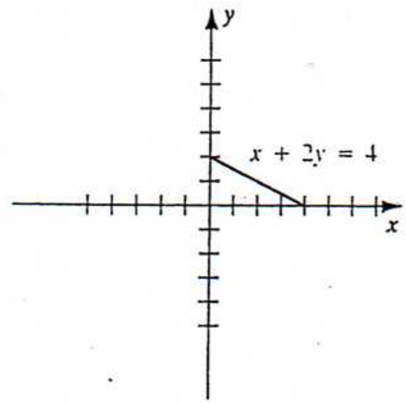
26 $y = \sqrt{x}$, $y = 0$, $x = 4$

(a) $x = 4$ (b) $x = 6$

(c) $y = 2$ (d) $y = 4$

Exer. 27–28: Set up an integral that can be used to find the volume of the solid generated by revolving the shaded region about the line (a) $y = -2$, (b) $y = 5$, (c [illegible] 7, and (d) $x = -4$.

27



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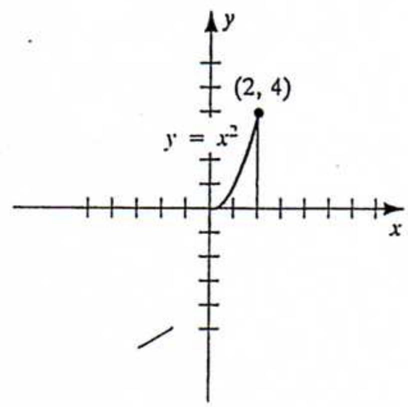

Exer. 29–34: Sketch the region R bounded by the graphs of the equations, and set up integrals that can be used to find the volume of the solid generated if R is revolved about the given line.

29 $y = x^3$, $y = 4x$; $y = 8$

30 $y = x^3$, $y = 4x$; $x = 4$

31 $x + y = 3$, $y + x^2 = 3$; $x = 2$

32 $y = 1 - x^2$, $x - y = 1$; $y = 3$

33 $x^2 + y^2 = 1$; $x = 5$

34 $y = x^{2/3}$, $y = x^2$; $y = -1$

Exer. 35–40: Use a definite integral to derive a formula for the volume of the indicated solid.

35 A right circular cylinder of altitude h and radius r

36 A cylindrical shell of altitude h, outer radius R, and inner radius r

37 A right circular cone of altitude h and base radius r

38 A sphere of radius r

39 A frustum of a right circular cone of altitude h, lower base radius R, and upper base radius r

40 A spherical segment of altitude h in a sphere of radius r

Solution to Swok Volume

5. $V = \pi\int_1^3 \left(\frac{1}{x}\right)^2 dx = \frac{2\pi}{3}$

7. $V = \pi\int_0^4 (x^2 - 4x)^2 dx = \frac{512\pi}{15}$

9. $V = \pi\int_0^2 (\sqrt{y})^2 dy = 2\pi$

11 $4y - y^2 = 0 \Rightarrow y = 0, 4.$ $4y - y^2 \ge 0$ on $[0, 4]$.

$V = \pi\int_0^4 (4y - y^2)^2 dy = \pi\left[\frac{16}{3}y^3 - 2y^4 + \frac{1}{5}y^5\right]_0^4 = \pi\left(\frac{1024}{3} - 512 + \frac{1024}{5}\right) = \frac{512\pi}{15}.$ 107.233

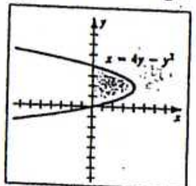

Figure 11

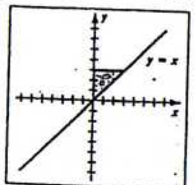

Figure 12

12 $V = \pi\int_0^3 (y)^2 dy = \pi\left[\frac{1}{3}y^3\right]_0^3 = 9\pi.$

13 $x^2 = 4 - x^2 \Rightarrow x = \pm\sqrt{2}.$ $4 - x^2 \ge x^2$ on $[-\sqrt{2}, \sqrt{2}]$. By (6.9), $V =$
$2\cdot\pi\int_0^{\sqrt{2}}\left[(4 - x^2)^2 - (x^2)^2\right]dx = 2\pi\left[16x - \frac{8}{3}x^3\right]_0^{\sqrt{2}} = 2\sqrt{2}\pi\left(16 - \frac{16}{3}\right) = \frac{64\pi\sqrt{2}}{3}.$

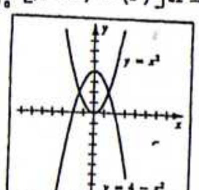

Figure 13

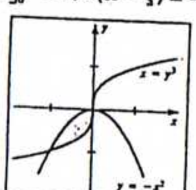

Figure 14

14 $x^{1/3} = -x^2 \Rightarrow x = -1, 0.$ Since the functions are both negative and $|x^{1/3}| \ge |-x^2|$ on $[-1, 0]$, $V = \pi\int_{-1}^0\left[(x^{1/3})^2 - (-x^2)^2\right]dx = \pi\left[\frac{3}{5}x^{5/3} - \frac{1}{5}x^5\right]_{-1}^0 = \frac{2\pi}{5}.$

15 $x = 4 - x \Rightarrow x = 2.$ $4 - x \ge x$ on $[0, 2]$.

$V = \pi\int_0^2\left[(4 - x)^2 - (x)^2\right]dx = 4\pi\left[4x - x^2\right]_0^2 = 4\pi(4) = 16\pi.$

17 $y^2 = 2y \Rightarrow y = 0, 2.$ $2y \ge y^2$ on $[0, 2]$.

$V = \pi\int_0^2\left[(2y)^2 - (y^2)^2\right]dy = \pi\left[\frac{4}{3}y^3 - \frac{1}{5}y^5\right]_0^2 = \pi\left(\frac{32}{3} - \frac{32}{5}\right) = \frac{64\pi}{15}.$

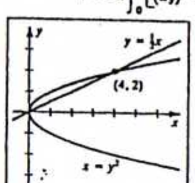

Figure 17

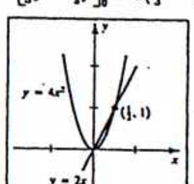

Figure 18

18 $x = \frac{1}{2}y$ and $x = \frac{1}{2}\sqrt{y}$ intersect when $y = 0, 1.$ $\frac{1}{2}\sqrt{y} \ge \frac{1}{2}y$ on $[0, 1]$.

$V = \pi\int_0^1\left[(\frac{1}{2}\sqrt{y})^2 - (\frac{1}{2}y)^2\right]dy = \frac{\pi}{4}\left[\frac{1}{2}y^2 - \frac{1}{3}y^3\right]_0^1 = \frac{\pi}{4}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\pi}{24}.$ .041π

19 $y^2 = y + 2 \Rightarrow y = -1, 2.$ $y + 2$ … $1, 2]$.

$V = \pi\int_{-1}^2\left[(y + 2)^2 - (y^2)^2\right]dy = \pi\left[\ldots\right)^3 - \frac{1}{5}y^5\Big]_{-1}^2 = \pi\left(\frac{224}{15} - \frac{8}{15}\right) = \frac{72\pi}{5}.$

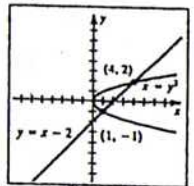

Figure 19

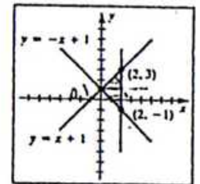

Figure 20

20 The lines intersect at (0, 1), (2, −1), (2, 3). If $-1 \le y \le 1$, the left boundary is $x = 1 - y$ and if $1 \le y \le 3$, it is $x = y - 1$. By symmetry,

$V = 2\cdot\pi\int_1^3\left[2^2 - (y - 1)^2\right]dy = 2\pi\left[4y - \frac{1}{3}(y - 1)^3\right]_1^3 = \frac{32\pi}{3}.$

21 $V = \pi\int_0^\pi(\sin 2x)^2 dx = \frac{\pi}{2}\int_0^\pi(1 - \cos 4x)\,dx = \frac{\pi}{2}\left[x - \frac{1}{4}\sin 4x\right]_0^\pi = \frac{1}{2}\pi^2.$

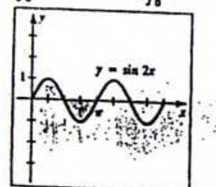

Figure 21

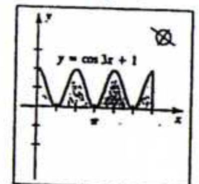

Figure 22

22 $V = \pi\int_0^{2\pi}(1 + \cos 3x)^2 dx = \pi\int_0^{2\pi}(1 + 2\cos 3x + \cos^2 3x)\,dx =$
$\pi\int_0^{2\pi}\left(1 + 2\cos 3x + \frac{1 + \cos 6x}{2}\right)dx = \pi\left[\frac{3}{2}x + \frac{2}{3}\sin 3x + \frac{1}{12}\sin 6x\right]_0^{2\pi} = 3\pi^2$

23 $V = \pi\int_0^{\pi/4}\left[(\cos x)^2 - (\sin x)^2\right]dx = \pi\int_0^{\pi/4}\cos 2x\,dx = \pi\left[\frac{1}{2}\sin 2x\right]_0^{\pi/4} = \frac{\pi}{2}.$

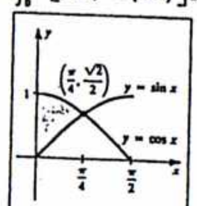

Figure 23

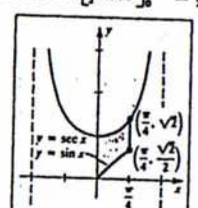

Figure 24

24 On $[0, \frac{\pi}{4}]$, $\sec x \ge \sin x$.

$V = \pi\int_0^{\pi/4}\left[(\sec x)^2 - (\sin x)^2\right]dx = \pi\int_0^{\pi/4}\left[\sec^2 x - \left(\frac{1 - \cos 2x}{2}\right)\right]dx =$
$\pi\left[\tan x - \frac{1}{2}x + \frac{1}{4}\sin 2x\right]_0^{\pi/4} = \pi\left(1 - \frac{\pi}{8} + \frac{1}{4}\right) = \frac{5\pi}{4} - \frac{1}{8}\pi^2.$

25 (a) The radius of a typical disk is $4 - x^2$.

$V = 2\cdot\pi\int_0^2(4 - x^2)^2 dx = 2\pi\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2 = \frac{512\pi}{15}.$

(b) The outer radius is $5 - x^2$ and the inner radius is $5 - 4$.

$V = 2\cdot\pi\int_0^2\left[(5 - x^2)^2 - (5 - 4)^2\right]dx = 2\pi\left[24x - \frac{10}{3}x^3 + \frac{1}{5}x^5\right]_0^2 = \frac{832\pi}{15}.$

27 (a) On the x-interval [0, 4], the outer radius is $(-\frac{1}{2}x + 2) - (-2)$ and the inner radius is $0 - (-2)$. $V = \pi\int_0^4\left\{\left[(-\frac{1}{2}x + 2) - (-2)\right]^2 - \left[0 - (-2)\right]^2\right\}dx$

(b) The outer radius is $5 - 0$ and the inner radius is $5 - (-\frac{1}{2}x + 2)$.

$V = \pi\int_0^4\left\{(5 - 0)^2 - \left[5 - (-\frac{1}{2}x + 2)\right]^2\right\}dx$

(c) On the y-interval [0, 2], the outer radius is $7 - 0$ and the inner radius is $7 - (-2y + 4)$. $V = \pi\int_0^2\left\{(7 - 0)^2 - \left[7 - (-2y + 4)\right]^2\right\}dy$.

(d) The outer radius is $(-2y + 4) - (-4)$ and the inner radius is $0 - (-4)$.

$V = \pi\int_0^2\left\{\left[(-2y + 4) - (-4)\right]^2 - \left[0 - (-4)\right]^2\right\}dy.$

28 (a) On the x-interval [0, 2], the outer radius is $x^2 - ($ … he inner radius is $0 - (-2)$. $V = \pi\int_0^2\{[x$ … $- [0 - (-2)]^2\}\,dx$.

(b) The outer radius is $5 - 0$ and the inner radius is $5 - x^2$.

$V = \pi\int_0^2\left[(5 - 0)^2 - (5 - x^2)^2\right]dx.$

(c) On the y-interval [0, 4], the outer radius is $7 - \sqrt{y}$ and the inner radius is $7 - 2$.

$V = \pi\int_0^4\left[(7 - \sqrt{y})^2 - (7 - 2)^2\right]dy.$

(d) The outer radius is $2 - (-4)$ and the inner radius is $\sqrt{y} - (-4)$.

$V = \pi\int_0^4\left\{\left[2 - (-4)\right]^2 - \left[\sqrt{y} - (-4)\right]^2\right\}dy.$

29 $x^3 = 4x \Rightarrow x = 0, \pm 2.$ For $x \le 0$, the radius of the outer disk is $(8 - 4x)$ and the radius of the inner disk is $(8 - x^3)$. For $x \ge 0$, the outer disk has radius $(8 - x^3)$ and the inner disk has radius $(8 - 4x)$.

$V = \pi\int_{-2}^0\left[(8 - 4x)^2 - (8 - x^3)^2\right]dx + \pi\int_0^2\left[(8 - x^3)^2 - (8 - 4x)^2\right]dx.$

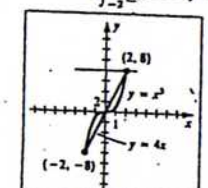

Figure 29

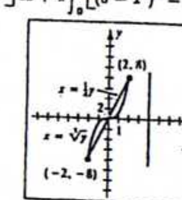

Figure 30

30 For $y \le 0$, the outer radius is $(4 - y^{1/3})$ and the inner radius is $(4 - \frac{1}{4}y)$.
For $y \ge 0$, the outer radius is $(4 - \frac{1}{4}y)$ and the inner radius is $(4 - y^{1/3})$.

$V = \pi\int_{-8}^0\left[(4 - y^{1/3})^2 - (4 - \frac{1}{4}y)^2\right]dy + \pi\int_0^8\left[(4 - \frac{1}{4}y)^2 - (4 - y^{1/3})^2\right]dy.$

31 The outer radius is $\left[2 - (3 - y)\right]$ and the inner radius is $(2 - \sqrt{3 - y})$.

$V = \pi\int_2^3\left\{\left[2 - (3 - y)\right]^2 - \left[2 - \sqrt{3 - y}\right]^2\right\}dy.$

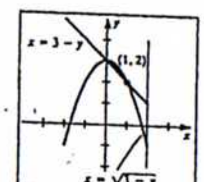

Figure 31

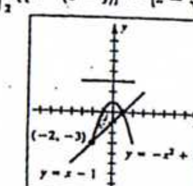

Figure 32

32 For $-2 \le x \le 1$, the outer radius is $\left[3 - (x - 1)\right]$ and the inner radius is $\left[3 - (1 - x^2)\right]$. $V = \pi\int_{-2}^1\left\{\left[3 - (x - 1)\right]^2 - \left[3 - (1 - x^2)\right]^2\right\}dx.$

33 For $-1 \le y \le 1$, the outer radius is $\left[5 - (-\sqrt{1 - y^2})\right]$ and the inner radius is $(5 - \sqrt{1 - y^2})$. $V = 2\cdot\pi\int_0^1\left\{\left[5 - (-\sqrt{1 - y^2})\right]^2 - \left[5 - \sqrt{1 - y^2}\right]^2\right\}dy.$

(c) For the revolution about the vertical line $x = 2$, the y-interval is $0 \le y \le 4$. The outer radius is $2 - (-\sqrt{y})$ and the inner radius is $2 - \sqrt{y}$.

$V = \pi\int_0^4\left\{\left[2 - (-\sqrt{y})\right]^2 - \left[2 - \sqrt{y}\right]^2\right\}dy = \pi\int_0^4 8\sqrt{y}\,dy = 8\pi\left[\frac{2}{3}y^{3/2}\right]_0^4 = \frac{128\pi}{3}$

(d) The outer radius is $3 - (-\sqrt{y})$ and the inner radius is $3 - \sqrt{y}$. $V =$
$\pi\int_0^4\left\{\left[3 - (-\sqrt{y})\right]^2 - \left[3 - \sqrt{y}\right]^2\right\}dy = \pi\int_0^4 12\sqrt{y}\,dy = 12\pi\left[\frac{2}{3}y^{3/2}\right]_0^4 = 64\pi$